

Problem 5: Limiting Behaviour Solution

Important observation!

One can prove these equations converge to a maximal independent set in the underlying graph. One can also prove that every maximal independent set is a stable fixed point of these equations.

Therefore, we can find the number of limit points by counting the number of maximal independent set in the underlying graph. In general, this problem is NP-complete. But since we consider circle graphs we can do this efficiently with a computer.

We are looking at configurations of the combinations 10 and 01, such that in total we end up with n numbers. So we are looking for x and y such that

$$2x + 3y = n.$$

We denote the function that returns this number of configurations by $f(n)$.

Note we have the following cases.

- We start with 1, end with 0. The number of cases is given by $f(n)$.
- We start with 01, end with 1. The number of cases is given by $f(n - 2)$.
- We start with 01, end with 10. The number of cases is given by $f(n - 3)$.
- We start with 001, end with 1. The number of cases is given by $f(n - 3)$.

So the total answer is given by $Total = f(n) + f(n - 2) + f(n - 3) + f(n - 3)$.

The function f can be calculated by a computer.